Exam 1 Review

Ex: let $f(x) = \frac{2}{x+3}$. find $f'(x)$

using the limit definition.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \left(\frac{\frac{2}{x+h+3} - \frac{2}{x+3}}{h} \right) \frac{(x+h+3)(x+3)}{(x+h+3)(x+3)}$$

$$= \lim_{h \to 0} \frac{2(x+3) - 2(x+h+3)}{h(x+h+3)(x+3)}$$

$$= \lim_{h \to 0} \frac{-2\cancel{h}}{\cancel{h}(x+h+3)(x+3)}$$

$$= \lim_{h \to 0} \frac{-2}{(x+h+3)(x+3)} = \boxed{\frac{-2}{(x+3)^2} = f'(x)}$$

Ex: let $g(x) = 3x^2 + 6x + 1$.

Simplify $\dfrac{g(x+h) - g(x)}{h}$.

$$\frac{g(x+h)-g(x)}{h} = \frac{3(x+h)^2 + 6(x+h) + 1 - [3x^2 + 6x + 1]}{h}$$

$$= \frac{\cancel{3x^2} + 6xh + 3h^2 + \cancel{6x} + 6h \cancel{+1} - \cancel{3x^2} - \cancel{6x} - \cancel{1}}{h}$$

$$= \frac{6xh + 3h^2 + 6h}{h}$$

$$= 6x + 3h + 6$$

Ex: find the equation of the tangent line to the graph of $f(x) = 4x^2 + 3x + 2$ at $x = 1$

Point $f(1) = 4(1)^2 + 3(1) + 2 = 9$ $\underline{(1,9)}$

Slope $f'(1)$

$f'(x) = 2(4)x + 3 = 8x + 3$

$f'(1) = 8(1) + 3 = 11$ $\underline{m = 11}$

tangent line

$\underline{y - 9 = 11(x - 1)}$

$\boxed{y = 11x - 2}$

Ex: Calculate the limits.

a) $\lim_{x \to 3} \frac{x+2}{x+1} = \frac{3+2}{3+1} = \boxed{\frac{5}{4}}$

b) $\lim_{x \to 0} 3x^2+1 = 3(0)^2+1 = \boxed{1}$

c) $\lim_{x \to 4} \frac{5+x}{x-4} \quad \frac{5+4}{4-4} = \frac{9}{0} = \boxed{\text{DNE}}$

d) $\lim_{x \to 2} \frac{x^2-3x+2}{x-2} \quad$ "$\frac{0}{0}$"

$= \lim_{x \to 2} \frac{\cancel{(x-2)}(x-1)}{\cancel{x-2}}$

$= \lim_{x \to 2} x-1 = 2-1 = \boxed{1}$

e) $\lim_{x\to 0} \dfrac{-x+x^2}{x-2x^2}$ "$\dfrac{0}{0}$"

$= \lim_{x\to 0} \dfrac{\cancel{x}(-1+x)}{\cancel{x}(1-2x)}$

$= \lim_{x\to 0} \dfrac{-1+x}{1-2x} = \dfrac{-1+0}{1-2(0)} = \boxed{-1}$

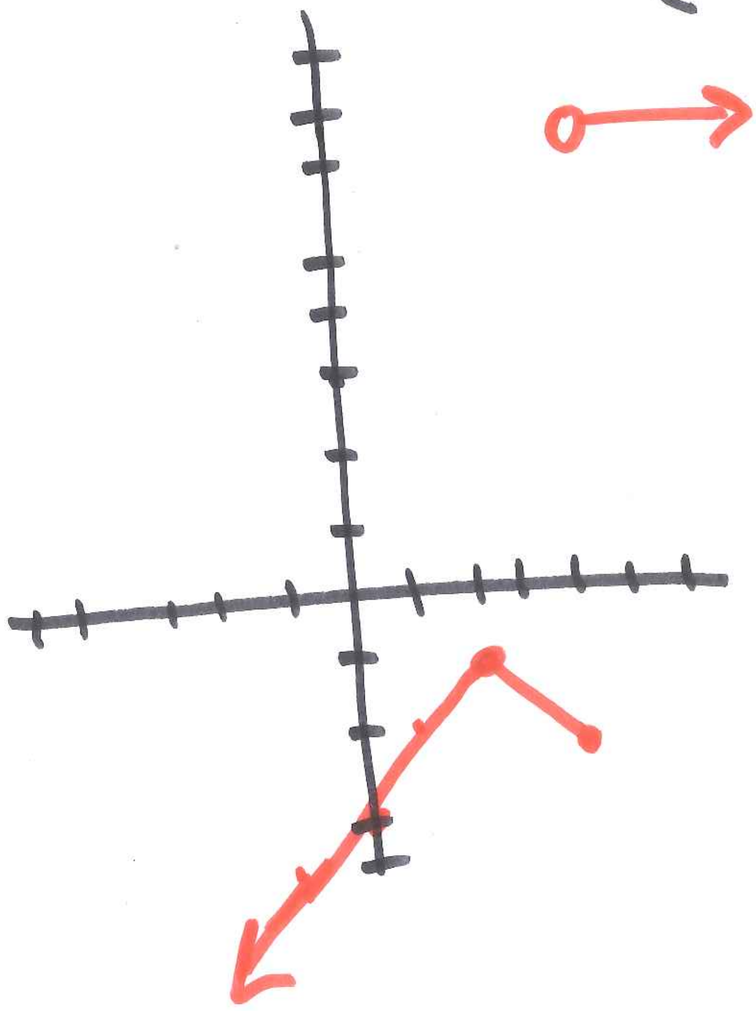
f) $\lim_{x\to\infty} \dfrac{3x^4}{x\cdot 4x^2}$

$= \lim_{x\to\infty} \dfrac{3x^4}{4x^3}$ * cancel x^3

$= \lim_{x\to\infty} \dfrac{3x}{4} =$ "$\dfrac{3\cdot\infty}{4}$"

$= \boxed{\text{DNE}}$

Ex: let $f(x) = \begin{cases} x-3 & x<2 \\ -\frac{1}{2}x & 2 \le x \le 4 \\ 7 & x>4 \end{cases}$



a) Where is $f(x)$ not continuous?

at $x=4$

b) Where is $f(x)$ not differentiable?

$x=2,4$

Ex: Simplify $\dfrac{\sqrt{x+h+1}-\sqrt{x+1}}{h}$.

$$\left(\frac{\sqrt{x+h+1}-\sqrt{x+1}}{h}\right)\left(\frac{\sqrt{x+h+1}+\sqrt{x+1}}{\sqrt{x+h+1}+\sqrt{x+1}}\right)$$

$$=\frac{(x+h+1)-(x+1)}{h\left(\sqrt{x+h+1}+\sqrt{x+1}\right)}$$

$$=\frac{\cancel{h}}{\cancel{h}\left(\sqrt{x+h+1}+\sqrt{x+1}\right)}$$

$$=\frac{1}{\left(\sqrt{x+h+1}+\sqrt{x+1}\right)}$$

$$\frac{2-\cancel{x}^{-1}}{\cancel{x}-2} = -1$$

Student Question

In general

$$\frac{a-b}{b-a} = -1$$

Ex:

$$\frac{2(\cancel{3-x})^{-1}}{(\cancel{x-3})(x+1)} = -\frac{2}{x+1}$$